Interpolation & Approximation

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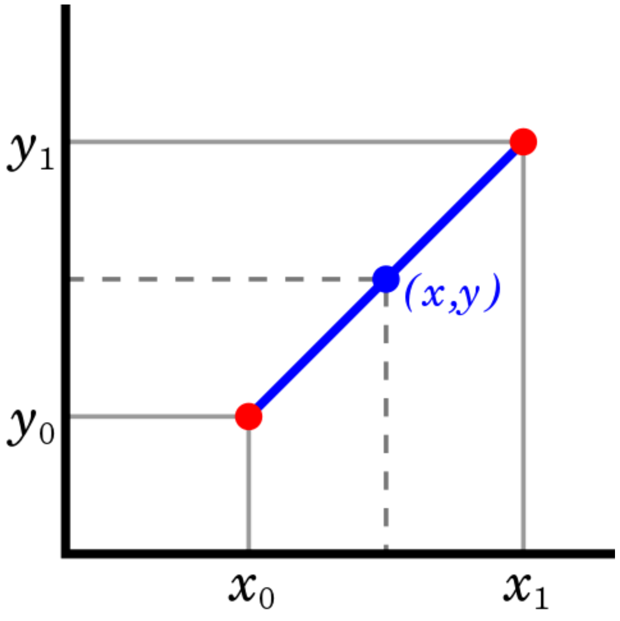
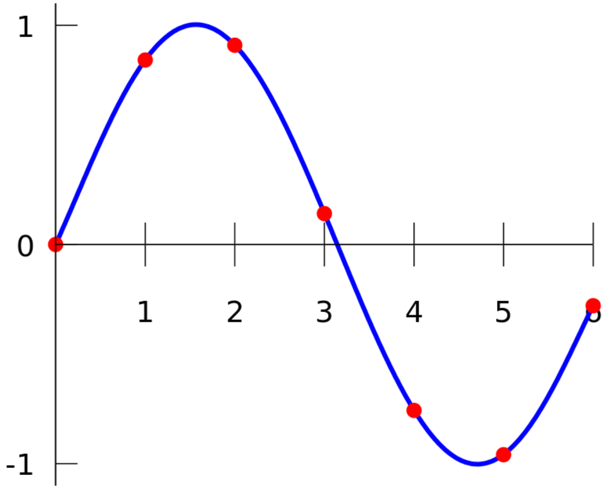
Unit 2 Interpolation and Approximation **8 Hrs.**

Introduction, Errors in Polynomial Interpolation, Lagrange's Polynomials, Newton's Interpolation using Difference and Divided Differences, Cubic Spline Interpolation, Least Squares Method for Linear and Non-linear Data.

The Concept of Interpolation

- **Interpolation is a family of methods studied in Numerical Analysis (NA), a field of Mathematics**. NA is oriented to finding approximate solutions to mathematical problems. Those solutions should be as accurate as possible.
- Let's assume that we have a set of points that correspond to a certain function. We don't know the exact expression of the function, but just those points. And we want to calculate the value of the function on some other points. Interpolation allows for approximating this function by using other simpler functions. **The approximated function should pass through all the points in the previously known set**. Then, it could be used to estimate other points not included in the set.
- Some of the types of Interpolation
  - Linear Interpolation
  - Polynomial Interpolation
  - Spline Interpolation



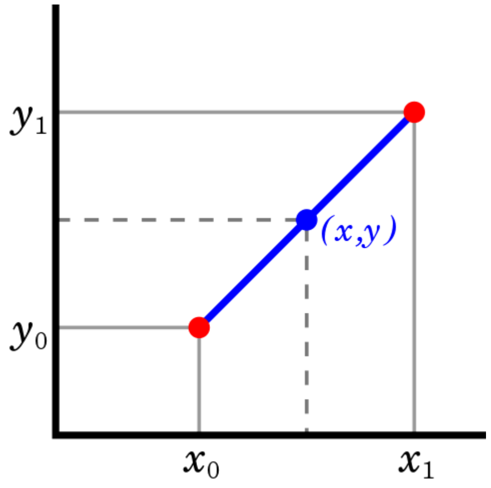


Linear Interpolation

Linear interpolation is a method used to interpolate between two points by a straight line. Let's consider two points (x_0, y_0), (x_1, y_1). Then the equation of the line passing by those points is *y = a + bx*, where:

$$a = \frac{y_0 . x_1 - y_1 . x_0}{x_1 - x_0}$$

$$b = \frac{y_1 - y_0}{x_1 - x_0}$$



Let's assume that we're approximating a value of some function *f* with a linear polynomial *P*. Then, the error *R* = *f(x)* - *P(x)* is calculated by:

$$|R| < \frac{(x_1 - x_0)^2}{8} . max(f''(x))$$

for *x* between the x_0 and x_1.

In the following figure, the blue line represents the linear interpolant between the two red points.

We can also use linear interpolation with more than two points, but the error increases according to the degree of curvature of the function.

Polynomial Interpolation

Polynomial interpolation is a method used to interpolate between N points by a polynomial. Let's consider a set with N points. Then we can always find a polynomial that passes through those N points. And this polynomial is unique. Polynomial interpolation is advantageous because polynomials are easier to evaluate, differentiate, and integrate.

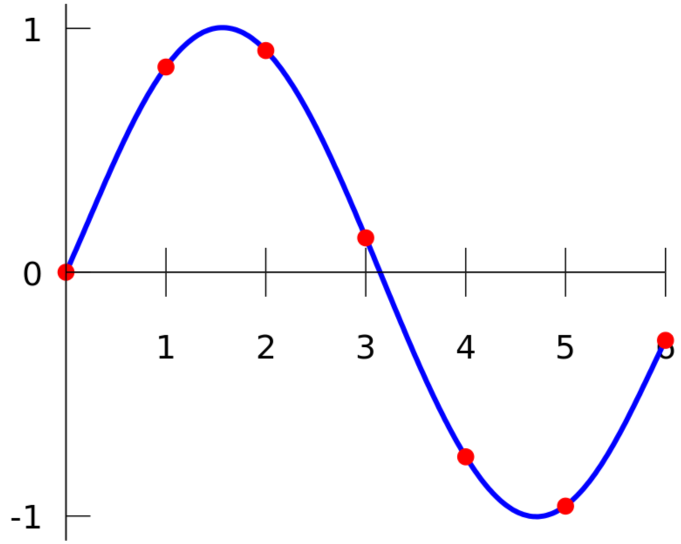
There are different approaches to calculating the coefficients of the interpolation polynomial. One of them is using the Lagrange polynomial defined as:

$$p(x) = \sum_{j=0}^{n} y_j . L_{n,j}(x)$$

Where

$$L_{n,j} = \prod_{k \neq j} \frac{x - x_k}{x_j - x_k}$$

The following blue curve shows the interpolation polynomial adjusted to the red dots.



Spline Interpolation

- Spline interpolation is a mathematical technique used to construct a smooth curve or surface that passes through a set of given points. It is particularly useful when you have a set of scattered data points and want to estimate values between those points or interpolate missing data.
- In spline interpolation, a spline function is used to connect the given points by a piecewise-defined curve or surface. The spline function is typically a polynomial function that is defined on each interval between two adjacent points. These polynomials are chosen in such a way that they smoothly join together at the given points, ensuring continuity of the curve or surface.
- The most commonly used form of spline interpolation is cubic spline interpolation, where the piecewise functions are cubic polynomials. The cubic spline interpolation provides a smooth curve that passes through each of the given points, and it also ensures the first and second derivatives of the curve are continuous at each point.
- Spline interpolation has applications in various fields such as computer graphics, numerical analysis, and data analysis. It provides a flexible and accurate way to approximate functions or estimate values based on a limited set of data points, while maintaining smoothness and continuity.

Interpolation vs Extrapolation

1. Range: Interpolation is used to estimate values within the known range of data points, while extrapolation is used to estimate values outside the known range.
2. Assumptions: Interpolation assumes that the relationship between the data points is continuous, while extrapolation assumes that the relationship observed within the known range will continue to hold true outside that range.
3. Uncertainty: Interpolation is generally considered to be more reliable because it is based on existing data. Extrapolation, on the other hand, comes with greater uncertainty and risk due to its reliance on assumptions about the data trend continuing.

In summary, interpolation and extrapolation are methods used to estimate values within or outside the range of known data points, respectively. Interpolation is used within the known range and assumes continuity, while extrapolation extends beyond the known range, making assumptions about the data trend continuing. Interpolation is generally more reliable, while extrapolation is riskier and should be used with caution.

Lagrange's Interpolation

Suppose $f(x)$ be a function with $f(x_0), f(x_1), f(x_2), \dots \dots \dots, f(x_n)$ corresponding to the values $x_0, x_1, x_2, \dots \dots \dots, x_n$ then the Lagrange's interpolation formula is given by;

$$f(x) = \frac{(x-x_1)(x-x_2).....(x-x_n)}{(x_0-x_1)(x_0-x_2).....(x_0-x_n)} \times f(x_0) + \frac{(x-x_0)(x-x_2).....(x-x_n)}{(x_1-x_1)(x_1-x_2).....(x_1-x_n)} \times f(x_1) + \dots\dots$$

$$\dots\dots\dots + \frac{(x-x_0)(x-x_1).....(x-x_{n-1})}{(x_n-x_0)(x_n-x_1).....(x_n-x_{n-1})} \times f(x_n)$$

Algorithm For Lagrange's interpolation

1. *Read the number of data 'n'.*
2. *Read the value at which value is needed (say y).*
3. *Read available data points x and f(x).*
4. *for i=0 to n*
   for j=0 to n
   if(i!=j)
   L[i]=L[i](y-x[j])/(x[i]-x[j])*
   end if
   end for
   end for
5. *for i=0 to n*
   *sum=sum+L[i]*f[i]*
   end for
6. *Print interpolation value 'sum' at y.*
7. *Stop*

Examples

1. ***Use Lagrange's interpolation formula find the value of f(x) at x= 10 i.e. f(10) from following data.***

x	*5*	*6*	*9*	*11*
f(x)	*12*	*13*	*14*	*16*

Sol^n:

Here, $x_0 = 5, x_1 = 6, x_2 = 9, x_3 = 11$

$$f(x_0) = 12, f(x_1) = 13, f(x_2) = 14, f(x_3) = 16$$

By Lagrange's interpolation; we have,

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \times f(x_0) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \times f(x_1) + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \times f(x_2) + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \times f(x_3)$$

$$\therefore f(10) = \frac{(10-6)(10-9)(10-11)}{(5-6)(5-9)(5-11)} \times 12 + \frac{(10-5)(10-9)(10-11)}{(6-5)(6-9)(6-11)} \times 13 + \frac{(10-5)(10-6)(10-11)}{(9-5)(9-6)(9-11)} \times 14 + \frac{(10-5)(10-6)(10-9)}{(11-5)(11-6)(11-9)} \times 16$$

= 2 - 4.3333 + 11.6666 + 5.3333

=14.6666

2. *Find the Lagrange interpolation polynomial to fit the following data.*

x_i	*0*	*1*	*2*	*3*
e^{x_i}	*0*	*1.7183*	*6.3891*	*19.0855*

Estimate the value of $e^{1.9}$.

Sol^n:

Here, $x_0 = 0, x_1 = 1, x_2 = 2, x_3 = 3$

$$f(x_0) = 0, f(x_1) = 1.7183, f(x_2) = 6.3891, f(x_3) = 19.0855$$

By Lagrange's interpolation; we have,

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \times f(x_0) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \times f(x_1) + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \times f(x_2) + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \times f(x_3)$$

$$or, f(x) = \frac{(x-1)(x-2)(x-3)}{(0-1)(0-2)(0-3)} \times 0 + \frac{(x-0)(x-2)(x-3)}{(1-0)(1-2)(1-3)} \times 1.7183 + \frac{(x-0)(x-1)(x-3)}{(2-0)(2-1)(2-3)} \times 6.3891 + \frac{(x-0)(x-1)(x-2)}{(3-0)(3-1)(3-2)} \times 19.0855$$

Now, to estimate the value of $e^{1.9}$,

$$\therefore f(1.9) = \frac{(1.9-1)(1.9-2)(1.9-3)}{(0-1)(0-2)(0-3)} \times 0 + \frac{(1.9-0)(1.9-2)(1.9-3)}{(1-0)(1-2)(1-3)} \times 1.7183 + \frac{(1.9-0)(1.9-1)(1.9-3)}{(2-0)(2-1)(2-3)} \times 6.3891 + \frac{(1.9-0)(1.9-1)(1.9-2)}{(3-0)(3-1)(3-2)} \times 19.0855$$

$= 0 + 0.1796 + 6.0089 – 0.5439$

$= 5.6446$

$\therefore e^{1.9} = 5.6446$

3. Find the value of $f(x)$ at $x = 1$ for the following data:

x	-1	-2	2	4
$f(x)$	-1	-9	11	69

<u>Soln:</u>

Here, $x_0 = -1, x_1 = -2, x_2 = 2, x_3 = 4$

$$f(x_0) = -1, f(x_1) = -9, f(x_2) = 11, f(x_3) = 69$$

By Lagrange's interpolation; we have,

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \times f(x_0) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \times f(x_1) + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \times f(x_2) + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \times f(x_3)$$

$$\therefore\ f(1) = \frac{(1+2)(1-2)(1-4)}{(-1+2)(-1-2)(-1-4)}(-1) + \frac{(1+1)(1-2)(1-4)}{(-2+1)(-2-2)(-2-4)}(-9) + \frac{(1+1)(1+2)(1-4)}{(2+1)(2+2)(2-4)}(11) + \frac{(1+1)(1+2)(1-2)}{(4+1)(4+2)(4-2)}(69)$$

$$= -0.6 + 2.25 + 8.25 - 6.9$$

$$= 3$$

$\therefore f(1) = 3$

C Source Code: Lagrange Interpolation

```c
#include<stdio.h>
#include<conio.h>

void main()
{
	float x[100], y[100], xp, yp=0, 
	int i,j,n;
	clrscr();
	/* Input Section */
	printf("Enter number of data: ")
	scanf("%d", &n);
	printf("Enter data:\n");
	for(i=1;i<=n;i++)
	{
		printf("x[%d] = ", i);
		scanf("%f", &x[i]);
		printf("y[%d] = ", i);
		scanf("%f", &y[i]);
	}
}
```

```c
printf("Enter interpolation point: ");
scanf("%f", &xp);
/* Implementing Lagrange Interpolation */
for(i=1;i<=n;i++)
{
	p=1;
	for(j=1;j<=n;j++)
	{
		if(i!=j)
		{
			p = p* (xp - x[j])/(x[i] - x[j]);
		}
	}
	yp = yp + p * y[i];
}
printf("Interpolated value at %.3f is %.3f.", xp, yp);
getch();
```

C Program Output: Lagrange Interpolation

```
Enter number of data: 5 ↲
Enter data:
x[1] = 5 ↲
y[1] = 150 ↲
x[2] = 7 ↲
y[2] = 392 ↲
x[3] = 11 ↲
y[3] = 1452 ↲
x[4] = 13 ↲
y[4] = 2366 ↲
x[5] = 17 ↲
y[5] = 5202 ↲
Enter interpolation point: 9 ↲
Interpolated value at 9.000 is 810.000.

Note: ↲ indicates ENTER is pressed.
```

Newton's Interpolation using divided differences

Suppose $f(x)$ be a function with $f(x_0), f(x_1), f(x_2), \dots\dots\dots, f(x_n)$ corresponding to the values $x_0, x_1, x_2, \dots\dots\dots, x_n$ then Newton divided difference interpolation is given by;

$$f(x) = f(x_0) + (x - x_0)f[x_0, x_1] + (x - x_0)(x - x_1)f[x_0, x_1, x_2] + \cdots \dots\dots\dots (x - x_0)(x - x_1)\dots..(x - x_n)f[x_0, x_1, \dots..x_n]$$

Or, it can be also written as;

$$f(x) = f_0 + (x - x_0)\Delta f_0 + (x - x_0)(x - x_1)\Delta^2 f_0 + (x - x_0)(x - x_1)(x - x_2)\Delta^3 f_0 + \cdots \dots$$

Now, we can construct the divided difference table as follows;

x	f	Δf	$\Delta^2 f$	$\Delta^3 f$
x_0	f_0			
		$\dfrac{f_1 - f_0}{x_1 - x_0} = \Delta f_0$		
x_1	f_1		$\dfrac{\Delta f_1 - \Delta f_0}{x_2 - x_0} = \Delta^2 f_0$	
		$\dfrac{f_2 - f_1}{x_2 - x_1} = \Delta f_1$		$\dfrac{\Delta^2 f_1 - \Delta^2 f_0}{x_3 - x_0} = \Delta^3 f_0$
x_2	f_2		$\dfrac{\Delta f_2 - \Delta f_1}{x_3 - x_1} = \Delta^2 f_1$	
		$\dfrac{f_3 - f_2}{x_3 - x_2} = \Delta f_2$		
x_3	f_3			

Algorithm for Newton Interpolation

1. *Read the number of data 'n'.*
2. *Read the value at which value is needed (say y).*
3. *Read the values of x_i and f_i.*
4. *Initialize sum=f_0, mult=1*
5. *for i=0 to n*

```
{
  for j=0 to n
  {
    fj=(fj+1-fj)/(xj+1-xj)
  }
  Repeat j
  Mult*=(y-xi)
  Sum+=fi*mult
}
Repeat i
```

6. *Print functional value 'sum' at y.*
7. *Stop*

Examples

1. Using the divide difference table (Newton's divided difference interpolation). Find the value of $f(1.75)$.

x	1.1	2.0	3.5	5	7.1
f	0.6981	1.4715	2.1287	2.0521	1.4480

The divide difference table for given data;

x	f	Δf	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$
1.1	0.6981				
		$\frac{1.4715-0.6981}{2.0-1.1}=0.8593$			
2.0	1.4715		$\frac{0.4381-0.8593}{3.5-1.1}=-0.1755$		
		$\frac{2.1287-1.4715}{3.5-2.0}=0.4381$		$\frac{-0.1630+0.1755}{5-1.1}=0.0031$	
3.5	2.1287		$\frac{-0.0510-0.4381}{5-2.0}=-0.1630$		$\frac{0.0190-0.0031}{7.1-1.1}=0.0026$
		$\frac{2.0521-2.1287}{5-3.5}=-0.0510$		$\frac{-0.0657+0.1630}{7.1-2.0}=0.0190$	
5	2.0521		$\frac{-0.2876+0.0510}{7.1-3.5}=-0.0657$		
		$\frac{1.4480-2.0521}{7.1-5}=-0.2876$			
7.1	1.4480				

We have,

$$f(x)=f(x_0)+(x-x_0)f[x_0,x_1]+(x-x_0)(x-x_1)f[x_0,x_1,x_2]+(x-x_0)(x-x_1)(x-x_2)f[x_0,x_1,x_2,x_3]+(x-x_0)(x-x_1)(x-x_2)(x-x_3)f[x_0,x_1,x_2,x_3,x_4]$$

$$f(1.175)=0.6981+(1.175-1.1)0.8593+(1.175-1.1)(1.175-2.0)(-0.1755)+(1.175-1.1)(1.175-2.0)(1.175-3.5)0.0031+(1.175-1.1)(1.175-2.0)(1.175-3.5)(1.175-5)0.0026$$

$$=0.7719$$

2. ***Using Newton's divide difference interpolating polynomial estimate the value of $f(x)$ at $x = 4$ for the function defined as***

x	*0*	*2*	*3*	*6*
f	*648*	*704*	*729*	*792*

The divide difference table for given data;

x	f	Δf	$\Delta^2 f$	$\Delta^3 f$
0	648			
		$\frac{704-648}{2-0} = 28$		
2	704		$\frac{25-28}{3-0} = -1$	
		$\frac{729-704}{3-2} = 25$		$\frac{-1+1}{6-0} = 0$
3	729		$\frac{21-25}{6-2} = -1$	
		$\frac{792-729}{6-3} = 21$		
6	792			

We have,

$$f(x) = f(x_0) + (x - x_0)f[x_0, x_1] + (x - x_0)(x - x_1)f[x_0, x_1, x_2] + (x - x_0)(x - x_1)(x - x_2)f[x_0, x_1, x_2, x_3]$$

$$= 648 + (x - 0)28 + (x - 0)(x - 2)(-1) + 0$$

$$\therefore f(4) = 648 + (4 - 0)28 + (4 - 0)(4 - 2)(-1)$$

$$= 752$$

<u>Methods to find the interpolation with equal intervals</u>

1. Newton Forward Interpolation method

2. Newton Backward Interpolation method

Newton Forward Interpolation method

We know that, Newton's Forward Interpolation formula as

$$y_s = y_0 + s\Delta y_0 + \frac{s(s-1)}{2!}\Delta^2 y_0 + \frac{s(s-1)(s-2)}{3!}\Delta^3 y_0 + \frac{s(s-1)(s-2)(s-3)}{4!}\Delta^4 y_0 + \cdots \ldots.$$

Where, $s = \frac{x_s - x_0}{h}$

x_s= Value at which interpolation is to be found

x_0= Initial value

h= Interval of 'x'

$y_0 = f(x_0), y_1 = f(x_1), \ldots\ldots, y_n = f(x_n)$

The difference table is:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
x_0	y_0			
		$\Delta y_0 = y_1 - y_0$		
x_1	y_1		$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$	
		$\Delta y_1 = y_2 - y_1$		$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$
x_2	y_2		$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$	
		$\Delta y_2 = y_3 - y_2$		
x_3	y_3			

Newton Backward Interpolation method

If the table is too long and if the required point is close to the end point of the table, we can use newton backward interpolation formula.

We know that, Newton's backward Interpolation formula as

$$y_s = y_n + s\nabla y_n + \frac{s(s+1)}{2!}\nabla^2 y_n + \frac{s(s+1)(s+2)}{3!}\nabla^3 y_n + \frac{s(s+1)(s+2)(s+3)}{4!}\nabla^4 y_n + \cdots ....$$

Where, $s = \frac{x_s - x_n}{h}$

x_s= Value at which interpolation is to be found

x_n= Final value

h= Interval of 'x'

The difference table is:

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$
x_0	y_0			
		$\nabla y_0 = y_1 - y_0$		
x_1	y_1		$\nabla^2 y_0 = \nabla y_1 - \nabla y_0$	
		$\nabla y_1 = y_2 - y_1$		$\nabla^3 y_0 = \nabla^2 y_1 - \nabla^2 y_0$
x_2	y_2		$\nabla^2 y_1 = \nabla y_2 - \nabla y_1$	
		$\nabla y_2 = y_3 - y_2$		
x_3	y_3			

1. ***Find the functional value at x = 25 from the following data using forward difference table.***

x	*10*	*20*	*30*	*40*	*50*
y	*0.1736*	*0.3420*	*0.5000*	*0.6428*	*0.7660*

Soln:

The difference table is;

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
10	0.1736				
		0.1684			
20	0.3420		-0.0104		
		0.1580		0.0048	
30	0.5000		-0.0152		-0.0004
		0.1428		0.0044	
40	0.6428		-0.0196		
		0.1232			
50	0.7660				

Here,

x_0=10

h=10

$$s = \frac{x_s - x_0}{h} = \frac{25-10}{10} = 1.5$$

Now according to Newton's forward difference formula;

$$y_s = y_0 + s\Delta y_0 + \frac{s(s-1)}{2!}\Delta^2 y_0 + \frac{s(s-1)(s-2)}{3!}\Delta^3 y_0 + \frac{s(s-1)(s-2)(s-3)}{4!}\Delta^4 y_0$$

$$y(25) = 0.1736 + (1.5)(0.1684) + \frac{1.5(1.5-1)}{2!}(-0.0104) + \frac{1.5(1.5-1)(1.5-2)}{3!}(0.0048) + \frac{1.5(1.5-1)(1.5-2)(1.5-3)}{4!}(-0.0004)$$

=0.4220

$\therefore y(25) = 0.4220$

2. *Construct Newton forward interpolation formula from given table to evaluate $f(5)$.*

x	*4*	*6*	*8*	*10*
y	*1*	*3*	*8*	*16*

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
4	1			
		2		
6	3		3	
		5		0
8	8		3	
		8		
10	16			

Here,

x_0=4

h=2

$s = \frac{x_s - x_0}{h} = \frac{5-4}{2} = 0.5$

Now according to Newton's forward difference formula;

$$y_s = y_0 + s\Delta y_0 + \frac{s(s-1)}{2!}\Delta^2 y_0 + \frac{s(s-1)(s-2)}{3!}\Delta^3 y_0$$

$$y_5 = 1 + (0.5)(2) + \frac{0.5(0.5-1)}{2!}(3) + \frac{0.5(0.5-1)(0.5-2)}{3!}(0)$$

= 1+ 1 – 0.375 + 0

= 1.625

$\therefore f(5) = 1.625$

3. ***Find the functional value at x = 3.6 from the following data using forward difference table.***

x	***2***	***2.5***	***3***	***3.5***	***4***	***4.5***
f(x)	***1.43***	***1.03***	***0.76***	***0.6***	***0.48***	***0.39***

Sol^n:

x	$y = f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
2	1.43					
		-0.4				
2.5	1.03		0.13			
		-0.27		-0.02		
3	0.76		0.11		-0.05	
		-0.16		-0.07		0.11
3.5	0.6		0.04		0.06	
		-0.12		-0.01		
4	0.48		0.03			
		-0.09				
4.5	0.39					

Here,
$x_0 = 2$
$h = 0.5$
$$s = \frac{x_s - x_0}{h} = \frac{3.6-2}{0.5} = 3.2$$

Now according to Newton's forward difference formula;

$$y_s = y_0 + s\Delta y_0 + \frac{s(s-1)}{2!}\Delta^2 y_0 + \frac{s(s-1)(s-2)}{3!}\Delta^3 y_0 + \frac{s(s-1)(s-2)(s-3)}{4!}\Delta^4 y_0 + \frac{s(s-1)(s-2)(s-3)(s-4)}{5!}\Delta^5 y_0$$

$$y_{3.6} = 1.43 + 3.2(-0.4) + \frac{3.2(3.2-1)}{2!}(0.13) + \frac{3.2(3.2-1)(3.2-2)}{3!}(-0.02) + \frac{3.2(3.2-1)(3.2-2)(3.2-3)}{4!}(-0.05) + \frac{3.2(3.2-1)(3.2-2)(3.2-3)(3.2-4)}{5!}(0.11)$$

= 1.43 – 1.28 + 0.4576 – 0.02816 – 0.00352 – 0.001239

= 0.574681

4. Find the value of f(x) at x=17 using Newton's backward interpolation method for the following data.

x	*0*	*5*	*10*	*15*	*20*
f(x)	***1.0***	***1.6***	***3.8***	***8.2***	***15.4***

Sol^n:

x	$y=f(x)$	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
0	1.0				
		0.6			
5	1.6		1.6		
		2.2		0.6	
10	3.8		2.2		0
		4.4		0.6	
15	8.2		2.8		
		7.2			
20	15.4				

Here,

$x_n = x_4 = 20$

$h=5$

$s = \frac{x_s - x_n}{h} = \frac{17-20}{5} = -0.6$

Now according to Newton's backward difference formula;

$$y_s = y_n + s\nabla y_n + \frac{s(s+1)}{2!}\nabla^2 y_n + \frac{s(s+1)(s+2)}{3!}\nabla^3 y_n + \frac{s(s+1)(s+2)(s+3)}{4!}\nabla^4 y_n$$

$$\therefore y_{17} = 15.4 + (-0.6)(7.2) + \frac{(-0.6)(-0.6+1)}{2!}(2.8) + \frac{(-0.6)(-0.6+1)(-0.6+2)}{3!}(0.6) + 0$$

$$= 15.4 - 4.32 - 0.336 - 0.0336 + 0$$

$$= 10.7104$$

$$\therefore y_{17} = 10.7104$$

5. Estimate the value of ln(3.5) using Newton's backward difference formula, given the following data

x	*1.0*	*2.0*	*3.0*	*4.0*
ln(x)	*0.0*	*0.6931*	*1.0986*	*1.3863*

x	y=*ln(x)*	∇y	$\nabla^2 y$	$\nabla^3 y$
1.0	0.0			
		0.6931		
2.0	0.6931		-0.2876	
		0.4055		0.1698
3.0	1.0986		-0.1178	
		0.2877		
4.0	1.3863			

x_n=x_3 $= 4.0$

h=1.0

$s = \frac{x_s - x_n}{h} = \frac{3.5-4.0}{1.0} = -0.5$

Now according to Newton's backward difference formula;

$$y_s = y_n + s\nabla y_n + \frac{s(s+1)}{2!}\nabla^2 y_n + \frac{s(s+1)(s+2)}{3!}\nabla^3 y_n$$

$$y_{3.5} = 1.3863 + (-0.5)(0.2877) + \frac{(-0.5)(-0.5+1)}{2!}(-0.1178) + \frac{(-0.5)(-0.5+1)(-0.5+2)}{3!}(0.1698)$$

$$= 1.3863 - 0.14385 + 0.014725 - 0.021225$$

$$= 1.23595$$

$\therefore \ln(3.5) = 1.23595$

Cubic Spline Interpolation

Cubic interpolation works by constructing the (cubic) polynomial in pieces. Given n points will construct n-1 different (cubic) polynomials. These polynomial have consistent derivatives at the end points.

Formula

Formula 1:

$$h_i a_{i-1} + 2a_i(h_i + h_{i+1}) + h_{i+1}a_{i+1} = 6\left[\frac{f_{i+1}-f_i}{h_{i+1}} - \frac{f_i - f_{i-1}}{h_i}\right] \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Where $a_0 = a_n = 0$

Formula 2:

$$s_i(x) = \frac{a_{i-1}}{6h_i}(h_i^2 U_i - U_i^3) + \frac{a_i}{6h_i}(U_{i-1}^3 - h_i^2 U_{i-1}) + \frac{1}{h_i}(f_i U_{i-1} - f_{i-1}U_i) \ldots\ldots\ldots\ldots (2)$$

$h_i = x_i - x_{i-1}$ and $U_i = x - x_i$

- Number of coefficient is equal to number of points.
- Evaluate equation (1) for $i = 1\ to\ n - 1$
- Now evaluate equation (2) at $i = a$ value given by position of interval.

Examples

1. Given the data points

x	4	9	16
f	2	3	4

Estimate the functional value f at x=7 using cubic splines.

Sol^n:

Here,

$h_1 = x_1 - x_0$=9-4=5

$h_2 = x_2 - x_1$=16-9=7

f_0=2, f_1=3, f_2=4

$a_0 = a_2$ =0

We have,

$$h_i a_{i-1} + 2a_i(h_i + h_{i+1}) + h_{i+1}a_{i+1} = 6\left[\frac{f_{i+1}-f_i}{h_{i+1}} - \frac{f_i - f_{i-1}}{h_i}\right]$$

For $i = 1$

$$h_1 a_0 + 2a_1(h_1 + h_2) + h_2 a_2 = 6\left[\frac{f_2 - f_1}{h_2} - \frac{f_1 - f_0}{h_1}\right]$$

$$0 + 2a_1(5+7) + 0 \;=\; 6\left[\frac{4-3}{7} - \frac{3-2}{5}\right]$$

$$24a_1 = 6\left[\frac{1}{7} - \frac{1}{5}\right]$$

$$\boldsymbol{a_1} = \mathbf{-0.0143}$$

Again from second formula,

$$s_i(x) = \frac{a_{i-1}}{6h_i}(h_i^2 U_i - U_i^3) + \frac{a_i}{6h_i}(U_{i-1}^3 - h_i^2 U_{i-1}) + \frac{1}{h_i}(f_i U_{i-1} - f_{i-1}U_i)$$

$$s_1(x) = \frac{a_0}{6h_1}(h_1^2 U_1 - U_1^3) + \frac{a_1}{6h_1}(U_0^3 - h_1^2 U_0) + \frac{1}{h_1}(f_1 U_0 - f_0 U_1)$$

$U_0 = x - x_0 = x - 4$

$U_1 = x - x_1 = x - 9$

$$\therefore s_1(7) = 0 + \frac{-0.0143}{6X5}[(7-4)^3 - 5^2(7-4)] + \frac{1}{5}[3(7-4) - 2(7-9)]$$

=2.6229

2. ***Estimate $f(3)$ from the following data using cubic spline interpolation.***

x	1	2.5	4	5.7
$f(x)$	-2.0	4.2	14.4	31.2

<u>Sol</u>

$h_1 = x_1 - x_0 = 2.5 - 1 = 1.5$
$h_2 = x_2 - x_1 = 4 - 2.5 = 1.5$
$h_3 = x_3 - x_2 = 5.7 - 4 = 1.7$
$f_0 = -2.0,\ f_1 = 4.2,\ f_2 = 14.4,\ f_3 = 31.2$
$a_0 = a_3 = 0$

We have,

$$h_i a_{i-1} + 2a_i(h_i + h_{i+1}) + h_{i+1}a_{i+1} = 6\left[\frac{f_{i+1}-f_i}{h_{i+1}} - \frac{f_i - f_{i-1}}{h_i}\right]$$

For $i = 1$,

$$h_1 a_0 + 2a_1(h_1 + h_2) + h_2 a_2 = 6\left[\frac{f_2-f_1}{h_2} - \frac{f_1-f_0}{h_1}\right]$$

$$0 + 2a_1(1.5+1.5) + 1.5a_2 = 6\left[\frac{14.4-4.2}{1.5} - \frac{4.2+2.0}{1.5}\right]$$

$$\boldsymbol{6a_1 + 1.5a_2 = 16} \ \textbf{............... (1)}$$

For $i = 2$,

$$h_2 a_1 + 2a_2(h_2 + h_3) + h_3 a_3 = 6\left[\frac{f_3-f_2}{h_3} - \frac{f_2-f_1}{h_2}\right]$$

$$1.5a_1 + 2a_2(1.5+1.7) + 0 = 6\left[\frac{31.2-14.4}{1.7} - \frac{14.4-4.2}{1.5}\right]$$

$$\boldsymbol{1.5a_1 + 6.4a_2 = 18.494} \ \textbf{............... (2)}$$

Solving equation (1) & (2) we get,

$a_1 = 2.065$
$a_2 = 2.406$

Now,

$$s_i(x) = \frac{a_{i-1}}{6h_i}(h_i^2 U_i - U_i^3) + \frac{a_i}{6h_i}(U_{i-1}^3 - h_i^2 U_{i-1}) + \frac{1}{h_i}(f_i U_{i-1} - f_{i-1}U_i)$$

$$s_2(x) = \frac{a_1}{6h_2}(h_2^2 U_2 - U_2^3) + \frac{a_2}{6h_2}(U_1^3 - h_2^2 U_1) + \frac{1}{h_2}(f_2 U_1 - f_1 U_2)$$

$$U_1 = x - x_1 = x - 2.5$$

$$U_2 = x - x_2 = x - 4$$

$$s_2(x) = \frac{2.065}{6\text{x}1.5}(1.5^2(x-4) - (x-4)^3) + \frac{2.406}{6\text{x}1.5}\left((x-2.5)^3 - 1.5^2(x-2.5)\right) + \frac{1}{1.5}(14.4(x-2.5) - 4.2(x-4))$$

$$\therefore s_2(3) = \frac{2.065}{9}(2.25(3-4) - (3-4)^3) + \frac{2.406}{9}\left((3-2.5)^3 - 2.25(3-2.5)\right) + \frac{1}{1.5}(14.4(3-2.5) - 4.2(3-4))$$

$$= 7.0793$$

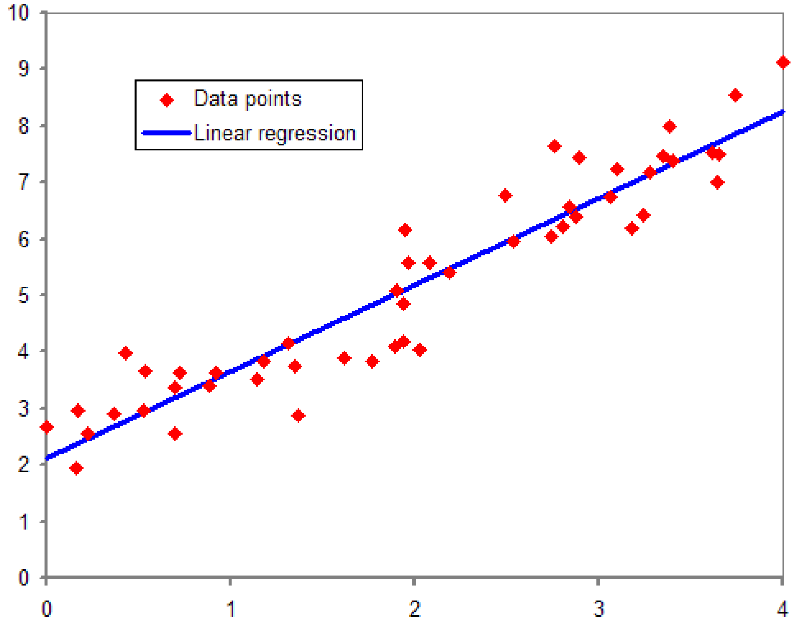
$$\therefore \boldsymbol{f(3) = 7.0793}$$

The Concept of Regression

- **Regression analysis is a family of methods in Statistics to determine relations in data**. **Those methods allow for evaluating the dependency of one variable on other variables**. They are used to find trends in the analyzed data and quantify them. Regression analysis attempt to precisely predict the value of a dependent variable from the values of the independent variables. It also addresses measuring the degree of impact of each independent variable in the result.
- The dependent variable is usually called the response or outcome. The independent variables are called features, and predictors, among others. This way, regression provides an equation to predict the response. For example, the value of the dependent variable is based on a particular combination of the feature values.
- regression analysis can be used for both interpolation and extrapolation, but extrapolation involves greater uncertainty and should be used with caution, taking into consideration the limitations and potential risks associated with extending the regression model beyond the observed data range.
- Some of the Regressions are:
  - Linear Regression
  - Least Square Method

Linear Regression

- **Linear regression approaches modeling the relationship between some dependent variables and other independent variables by a linear function**. The linear regression model assumes a linear relationship between the dependent variable *y* and the independent variable *x*, like in the equation $y = a + b.x$, where *y* is the estimated dependent variable, *a* is a constant, and *b* is the regression coefficient, and *x* is the independent variable.
- Linear regression considers that both variables are linearly related on average. Thus, for a fixed value of *x*, the actual value of *y* differs by a random amount from its expected value. Therefore, there is a random error ε, and the equation should be expressed as $y = a + bx + \varepsilon$, where ε is a random variable.



The actual values of *a* and *b* are unknown. Therefore, we need to estimate them from the sample data consisting of n observed pairs $(x_1, y_1), \ldots, (x_n, y_n)$:

Next, the blue line represents the regression model corresponding to the red points.

Least Squares Method

The method of least squares **allows estimating the values of *a* and *b* in the equation** $y = a + b.x$ **while minimizing the random error**. Let's assume that $\varepsilon_i = y_i - \hat{y}_i$ for i = 1, …, n, where y_i: the observed value, $\hat{y}_i$ is the estimated value and ε_i is the residual between both values. The least-squares method minimizes the sum of the squared residuals between the observed and the estimated values.

The slope coefficient b is calculated by:

$$b = \frac{S_{xy}}{S_{xx}}$$

where:

$$S_{xy} = \sum(x_i.y_i) - \frac{(\sum x_i)(\sum y_i)}{n}$$

$$S_{xx} = \sum x_i^2 - \frac{(\sum x_i)^2}{n}$$

The intercept *a* is calculated by:

$$a = \frac{\sum y_i}{n} - b.\frac{\sum x_i}{n}$$

Similarities Between Interpolation & Regression

Interpolation and regression methods present similarities as they both:

- Derive from Mathematics
- Are oriented to process a set of data points
- Are used to calculate new points
- New calculated points should be accurate as possible
- Model the dataset by a function
- Have models for linear and non-linear cases

Difference Between Interpolation & Regression

Interpolation	Regression
Comes from Numerical Analysis	Comes from Statistics
Assumes that points in the dataset represent the values of a function accurately	Accepts inexact values
Assumes that one value of the independent variable just could be associated with one value of the dependent one	Accepts that the same value of the independent variable could have several values of the dependent variable associated
The approximated function should match exactly with all the dataset points	The approximated function does not have to match exactly with the dataset points
The error of the estimated values is bounded by some specific expression	The error of the estimated values is bounded on average
The approximated functions are frequently used for numerical integration and differentiation	The approximated functions are frequently used for prediction and forecasting

Curve Fitting

Curve fitting is the process of introducing mathematical relationship between dependent and independent variables in the form of an equation for a given set of data.

Fitting of a straight line

$y = a + bx$ ……… (i) where a and b are constant and are unknown.

The normal equation of (i) is

$\sum y_i = na + b\sum x_i$ …….. (ii)

$\sum x_i y_i = a\sum x_i + b\sum x_i^2$ …….. (iii)

Solving for a and b, we get

$$\boxed{\begin{gathered} b = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - \left(\sum x_i\right)^2} \\ \\ a = \frac{\sum y_i}{n} - b\frac{\sum x_i}{n} = \bar{y} - b\bar{x} \end{gathered}}$$

Algorithm for linear Regression

1. *Read values of n, x_i, y_i*
2. *Compute sum of powers and products*
   $\sum x_i$, $\sum y_i$, $\sum x_i^2$, $\sum x_i y_{i_i}$
3. *Compute:*

   $$b = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - \left(\sum x_i\right)^2} \quad \textit{and}$$

   $$a = \bar{y} - b\bar{x}$$
4. *Check whether the denominator of the equation for b is zero. If not compute a and b.*
5. *Print out the equation.*
6. *Stop*

Examples

1. ***Fit a straight line to the following set of data.***

x	*1*	*2*	*3*	*4*	*5*
y	*3*	*4*	*5*	*6*	*8*

x	y	x^2	xy
1	3	1	3
2	4	4	8
3	5	9	15
4	6	16	24
5	8	25	40
$\sum x = 15$	$\sum y = 26$	$\sum x^2 = 55$	$\sum xy = 90$

We have,

$$b = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - \left(\sum x_i\right)^2} = \frac{5\times 90 - 15\times 26}{5\times 55 - 15^2} = 1.2$$

$$a = \frac{\sum y_i}{n} - b\frac{\sum x_i}{n} = \frac{26}{5} - 1.2\times\frac{15}{5} = 1.6$$

Therefore the linear equation is

$$y = 1.6 + 1.2x$$

2. Given the data set (x_i, y_i) as (20.5, 765), (32.7, 826),(51.0, 873), (73.2, 942), (95.7, 1032) find the linear least square to fit given data.

<u>Sol^n:</u>

x	y	x^2	xy
20.5	765	420.25	15682.5
32.7	826	1069.29	27010.2
51.0	873	2601	44523
73.2	942	5358.24	68954.4
95.7	1032	9158.49	98762.4
$\sum x = 273.1$	$\sum y = 4438$	$\sum x^2 = 18607.27$	$\sum xy = 254932.5$

We have,

$$b = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - \left(\sum x_i\right)^2} = \frac{5 \times 254932.5 - 273.1 \times 4438}{5 \times 18607.27 - (273.1)^2} = 3.3949$$

$$a = \frac{\sum y_i}{n} - b\frac{\sum x_i}{n} = \frac{4438}{5} - 3.3949 \times \frac{273.1}{5} = 702.171$$

Therefore the linear equation is

$$y = 702.171 + 3.3949x$$

3. *Fit the following set of data to a curve of the form $y = ab^x$.*

x	2	4	6	8	10	12
y	16	17.1	8.7	6.4	4.7	2.6

Given that,

$y = ab^x$ …….. (i)

Taking log on both side; we have,

$\log y = \log a + xlogb$ ………. (ii)

Comparing equation (ii) with $Y = A + Bx$

$Y = \log y$

$B = \log b$

$A = \log a$

X	y	Y=logy	X^2	XY
2	16	1.20412	4	2.40824
4	11.1	1.04532	16	4.18128
6	8.7	0.93952	36	5.63712
8	6.4	0.80618	64	6.44944
10	4.7	0.67209	100	6.7209
12	2.6	0.41497	144	4.97964
$\sum X = 42$		$\sum Y = 5.0822$	$\sum X^2 = 364$	$\sum XY = 30.37662$

We have,

$$B = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - \left(\sum x_i\right)^2} = \frac{6 \times 30.37662 - 42 \times 5.0822}{6 \times 364 - 42^2} = -0.07427$$

$$A = \frac{\sum y_i}{n} - B\frac{\sum x_i}{n} = \frac{5.0822}{6} - (-0.07427) \times \frac{42}{6} = 1.36692$$

Now,

$a = antilog(1.36692) = 23.27662$

$b = antilog(-0.07427) = 0.84281$

So the equation (i) becomes

$$y = 23.27662 \times 0.84281^x$$

4. *Fit the following set of data to a curve of the form $y = ae^{bx}$.*

x	2	4	6	8	10	12
y	16	17.1	8.7	6.4	4.7	2.6

Sol^n:

The given curve is $y = ae^{bx}$ ……….. (i)

Taking log on both sides, we have,

$logy = loga + bxloge$ ……….. (ii)

Comparing equation (ii) with $Y = A + Bx$

$Y = \log y$

$B = b \log e$

$A = \log a$

X	y	Y=logy	X^2	XY
2	16	1.20412	4	2.40824
4	11.1	1.04532	16	4.18128
6	8.7	0.93952	36	5.63712
8	6.4	0.80618	64	6.44944
10	4.7	0.67209	100	6.7209
12	2.6	0.41497	144	4.97964
$\sum X = 42$		$\sum Y = 5.0822$	$\sum X^2 = 364$	$\sum XY = 30.37662$

We have,

$$B = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - \left(\sum x_i\right)^2} = \frac{6\times 30.37662 - 42\times 5.0822}{6\times 364 - 42^2} = -0.07427$$

$$A = \frac{\sum y_i}{n} - B\frac{\sum x_i}{n} = \frac{5.0822}{6} - (-0.07427)\times\frac{42}{6} = 1.36692$$

Now,

$$a = \ antilog(1.36692) = 23.27662$$

$$b = \frac{B}{loge} = \frac{-0.07427}{loge} = -0.17101$$

The equation (i) becomes

$$y = 23.27662e^{-0.17101x}$$

Fitting quadratic polynomial

The equation for quadratic polynomial is

$y = a_1 + a_2 x + a_3 x^2$ .... (i)

The normal form equation of (i) are

$na_1 + a_2 \sum x_i + a_3 \sum x_i^2 = \sum y_i$ .... (ii)

$a_1 \sum x_i + a_2 \sum x_i^2 + \ a_3 \sum x_i^3 = \sum x_i y_i$ .... (iii)

$a_1 \sum x_i^2 + a_2 \sum x_i^3 + a_3 \sum x_i^4 = \sum x_i^2 y_i$ .... (iv)

These equation can be expressed in matrix form as

$$\begin{bmatrix} n & \sum x_i & \sum x_i^2 \\ \sum x_i & \sum x_i^2 & \sum x_i^3 \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum x_i y_i \\ \sum x_i^2 y_i \end{bmatrix}$$

Examples

1. Find the best fitting quadratic polynomial from following data using least square approximation.

x	*-2*	*-1.2*	*0*	*1*	*1.2*	*2.5*	*3*	*4.5*	*6.3*
y	***10.39***	***2.96***	***-2.0***	***-2.63***	***-2.46***	***0.83***	***3.1***	***12.8***	***30.4***

Given no. of data $(n) = 9$

The equation for quadratic polynomial is

$y = a_1 + a_2x + a_3x^2$......... (i)

The given equation in regression matrix form is

$$\begin{bmatrix} n & \sum x & \sum x_i^2 \\ \sum x_i & \sum x_i^2 & \sum x_i^3 \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum x_i y_i \\ \sum x_i^2 y_i \end{bmatrix} \text{.......... (ii)}$$

The various summations are given below,

$\sum x_i = 19.3$

$\sum y_i = 53.39$

$\sum x_i^2 = 86.07$

$\sum x_i^3 = 376.79$

$\sum x_i^4 = 708.5683$

$\sum x_i y_i = 230.5810$

$\sum x_i^2 y_i = 1538.5135$

Note: To find the value of $\sum x_i, \sum y_i, \sum x_i^2, \sum x_i^3, \sum x_i^4, \sum x_i y_i,$ $\sum x_i^2 y_i$ using calculator (CASIO fx-991ES) perform following steps:

1. Press→(mode) and 3
2. Select 3 and put the value of *x* and *y* and press AC.
3. Press :(shift) and (stat i.e. 1)
4. Select 3: Sum and take the required value.

Now from (ii)

$$\begin{bmatrix} 9 & 19.3 & 86.07 \\ 19.3 & 86.07 & 376.79 \\ 86.07 & 376.79 & 708.5683 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 53.39 \\ 230.5810 \\ 1538.5135 \end{bmatrix}$$

i.e.

$9a_1 + 19.3a_2 + 86.07a_3 = 53.39$

$19.3a_1 + 86.07a_2 + 376.79a_3 = 230.5810$

$86.07a_1 + 376.79a_2 + 708.5683a_3 = 1538.5135$

Solving, we get {You can use calculator to solve these equations.

$a_1 = 0.5506$, $a_2 = 5.0132$, $a_3 = -0.5614$

The equation (i) becomes

$$y = 0.5506 + 5.0132x - 0.5614x^2$$

Which is required quadratic polynomial.

2. *Fit a quadratic polynomial to the following set of data using least square approximation.*

x	*1*	*2*	*3*	*4*	*5*
y	*10*	*12*	*8*	*10*	*14*

Given no. of data (n) =5

The equation for quadratic polynomial is

$y = a_1 + a_2x + a_3x^2$......... (i)

The given equation in regression matrix form is

$$\begin{bmatrix} n & \sum x & \sum x_i^2 \\ \sum x_i & \sum x_i^2 & \sum x_i^3 \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum x_i y_i \\ \sum x_i^2 y_i \end{bmatrix} \text{.......... (ii)}$$

The various summations are given below,

$\sum x_i = 15$, $\sum y_i = 54$, $\sum x_i^2 = 55$,

$\sum x_i^3 = 225$, $\sum x_i^4 = 979$, $\sum x_i y_i = 168$, $\sum x_i^2 y_i = 640$

Now from (ii)

$$\begin{bmatrix} 5 & 15 & 55 \\ 15 & 55 & 225 \\ 55 & 225 & 979 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 54 \\ 168 \\ 640 \end{bmatrix}$$

i.e.

Now from (ii)

$$\begin{bmatrix} 5 & 15 & 55 \\ 15 & 55 & 225 \\ 55 & 225 & 979 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 54 \\ 168 \\ 640 \end{bmatrix}$$

i.e.

$5a_1 + 15a_2 + 55a_3 = 54$
$15a_1 + 55a_2 + 225a_3 = 168$
$55a_1 + 225a_2 + 979a_3 = 640$

Solving, we get

$a_1 = 13$, $a_2 = -3.686$, $a_3 = 0.714$

The equation (i) becomes

$$y = 13 - 3.686x + 0.714x^2$$

Which is required quadratic polynomial.

Lab Questions

6. Write a program to demonstrate Lagrange’ Polynomial
7. Write a C program for Curve Fitting (Fitting Linear Equations)

Source Code for Linear Curve Fitting in C:

```
#include<stdio.h>
#include<conio.h>
#include<math.h>
int main()
{

    int n,i,x[20],y[20],sumx=0,sumy=0,sumxy=0,sumx2=0;
    float a,b;
    printf("\n   C program for Linear Curve Fitting \n ");
    printf("\n Enter the value of number of terms n:");
    scanf("%d",&n);
    printf("\n Enter the values of x:\n");
    for(i=0;i<=n-1;i++)
    {
        scanf(" %d",&x[i]);

    }
    printf("\n Enter the values of y:");
    for(i=0;i<=n-1;i++)
    {
        scanf("%d",&y[i]);
    }
    for(i=0;i<=n-1;i++)
    {
        sumx=sumx +x[i];
        sumx2=sumx2 +x[i]*x[i];
        sumy=sumy +y[i];
        sumxy=sumxy +x[i]*y[i];

    }
    a=((sumx2*sumy -sumx*sumxy)*1.0/(n*sumx2-sumx*sumx)*1.0);
    b=((n*sumxy-sumx*sumy)*1.0/(n*sumx2-sumx*sumx)*1.0);
    printf("\n\nThe line is Y=%3.3f +%3.3f X",a,b);
    return(0);
}
```

```
   C program for Linear Curve Fitting

 Enter the value of number of terms n:4

 Enter the values of x:
2
3
4
5

 Enter the values of y:5
7
8
11


The line is Y=1.100 +1.900 X
Process returned 0 (0x0)   execution time : 19.395 s
Press any key to continue.
```

Source Code for Exponential Curve Fitting in C:

```
#include<stdio.h>
#include<conio.h>
#include<math.h>
int main()
{

    int n,i;
    float Y[20],sumx=0,sumy=0,sumxy=0,sumx2=0,x[20],y[20];
    float a,b,A;
    printf("\n  C program for Exponential Curve fitting\n");
    printf("\n Enter the value of number of terms n:");
    scanf("%d",&n);
    printf("\n Enter the values of x:");
```

```
for(i=0;i<=n-1;i++)
{
    scanf("%f",&x[i]);

}
printf("\n Enter the values of y:");
for(i=0;i<=n-1;i++)
{
    scanf("%f",&y[i]);
}
```

```
for(i=0;i<=n-1;i++)
{
    Y[i]=log(y[i]);
}
for(i=0;i<=n-1;i++)
{
    sumx=sumx +x[i];
    sumx2=sumx2 +x[i]*x[i];
    sumy=sumy +Y[i];
    sumxy=sumxy +x[i]*Y[i];

}
A=((sumx2*sumy -sumx*sumxy)*1.0/(n*sumx2-sumx*sumx)*1.0);
b=((n*sumxy-sumx*sumy)*1.0/(n*sumx2-sumx*sumx)*1.0);
a=exp(A);
printf("\n\n The curve is Y= %4.3fe^%4.3fX",a,b);
return(0);
```

```
 C program for Exponential Curve fitting

Enter the value of number of terms n:4

Enter the values of x:4
8
12
16

Enter the values of y:6
9
12
17


The curve is Y= 4.366e^0.085X
Process returned 0 (0x0)   execution time : 32.461 s
Press any key to continue.
```

Past Questions

2. Write an algorithm and a program to compute the interpolation value at a specific point, given a set of data points, using Lagrange interpolation method.

show solution
asked in Model Question

2. How polynomial interpolation differs with Cubic Spline interpolation? Explain. Find the best fit curve of quadratic polynomial using least square approximation from following data.

asked in 2075(Old Course)

x	-2.1	-1.5	1.2	2.5	3.6	4.5
$f(x)$	3.1	2.2	6.8	14.4	23.1	32.2

6. What is Newton's interpolation? Obtain the divided difference table from the following data set and estimate the *f(x)* at *x=2* and *x=5*.

asked in 2078

x	3.2	2.7	1.0	4.8	5.6
$f(x)$	22.0	17.8	14.2	38.3	51.7

6. Estimate the value of ln(3.5) using Newton's backward difference formula, given the following data

show solution
asked in Model Question

x	1.0	2.0	3.0	4.0
ln	0.0	0.6931	1.0986	1.3863

4. Find the square root of 3.5 using second order Larange interpolation polynomial using the following data table.

X	1	2	3	4	5
F(x)	1	1.4142	1.7321	2	2.2361

10 (a) Fit a straight line to the following set of data points.

X	1	3	4	6	8	9
y	1	3	4	4	5	7

11. Given the data points:

I	0	1	2
Xi	2	3	4
fi	4	9	16

Estimate the function value f at x = 2-5 using cubic splines.